Archimedes' Principle, Buoyancy, and Density

Equipment

- Chemical splash goggles (Students bring their own)
- Distilled/Deionized Water, Isopropyl alcohol
- Computer with a spreadsheet software
- Set of Digital Calipers
- Force Sensor
- Plastic bins to catch overflow.
- Graduated cylinder
- Aluminum Container with and without spout
- Vertical stand, perpendicular clamp, horizontal rod (between 20 cm and 50 cm)
- Metal Ball with a string attached to it
- Wooden cylinder with pencil lines marking off equal lengths

Objectives

- *Verify Archimedes' principle and use it to determine the density of a given liquid.*

Introduction

The famous legend tells us that Archimedes was the person who discovered that *the volume of displaced water equals the volume of a submerged object*. He came up with that idea as he was trying to measure the volume of a crown of unusual shape. Puzzled he had filled his bathtub flush with water and water overflowed when he got inside of the tub. The idea that the amount of water splashed out of the tub is exactly the volume of his own body struck him and he ran outside of his house crying "Eureka!" This means, "I have found it".

Archimedes' Principle itself isn't directly about volume, it's about buoyancy. It states that the buoyant upward force acting on an object entirely or partially submerged in a fluid is equal to the weight of the fluid displaced by the object.

For a given object, the weight can be directly calculated from the mass or from the density and volume:

$$F_g = mg = \rho V g$$

The buoyant force is found by applying the same idea to the fluid instead of the object:

$$F_B = m_{\text{fluid}} g = \rho_{\text{fluid}} V_{\text{displaced}} g \tag{1}$$

Here, m_{fluid} is the mass of the displaced fluid, which is broken down as the density of the fluid ρ_{fluid} multiplied by the submerged volume of the object $V_{\text{displaced}}$.

For a prism-shaped object like a cylinder, the submerged volume is equal to the cross-sectional area, A, multiplied by the submerged depth, d. So the buoyant force can be written as:

$$F_B = \rho_{\text{fluid}} A d g \quad (2)$$

If the object is lowered into the fluid while the buoyant force is measured, the slope of the graph of F_B versus d is proportional to the density of the fluid.

Part 1. Volume of the Displaced Liquid

The purpose of this experiment is to verify Archimedes' "finding" that the volume of the displaced liquid is the same as the volume of the object immersed. A metal ball will be used as the solid object.

1. Find the volume of the ball by measuring its diameter and using that to calculate the volume of the ball as if it is a perfect sphere.
2. Submerge the ball in water and determine the volume of the water displaced. (See Figure 1.)

- Place the aluminum container (the one with a spout) in position where you can catch any overflow with the graduated cylinder.
- Fill the container with water so it just overflows (do not catch this water), then allow it to stop dripping. Note that if you move the container after it is full, you'll slosh some water out, so get the container fixed in position first, then fill it.
- Prepare to catch any additional water that comes out of the spout with the graduated cylinder.
- Lower the ball in the water while catching the overflow in the graduated cylinder.

Figure 1. Ready to perform Part 1. The upper container (with the spout) was set in place, and then it was "topped off" with water. (The extra water fell into the plastic bin.) The graduated cylinder is ready to catch the overflow that will come out when the brass ball is lowered into the water. (Do not copy our picture into your lab report!)

3. Measure the displaced water volume using the graduated cylinder. **Keep this cylinder of water for Experiment 2!**
4. Compare the results. If you're far off, try it again, being more careful not to lose water before submersing the ball. (Note: 1 cm^3 = 1 ml)

Ball diameter (cm)	
V_{ball} (cm^3)	
V_{water} (ml)	
% Difference	

Table 1: Replace this text with an appropriate caption.

Part 2. Weight of the Displaced Liquid

The purpose of this experiment is to verify Archimedes' Principle for a submerged object. This principle states that the buoyant force acting on a submerged object equals the weight of displaced liquid.

1. Open Capstone File "Force Sensor – Digits Display".
2. Suspend the ball from the hook of the Force Sensor and record its weight in air.
3. Bring a cup of water up from underneath the ball. (Figure 3)
4. Make sure the ball isn't touching the side or bottom of the cup. While the ball is submerged, record the weight of the ball in water.
5. Using one of the digital balances, find the mass of the graduated cylinder from Experiment 1 with and without the water.
6. Calculate the buoyancy force as the difference of the weight of the ball in air and in water.
7. Calculate the mass of the displaced water as the difference in mass of the graduated cylinder with and without water. ***Convert the final value to kilograms.***
8. Calculate the weight of the displaced water.
9. Compare the experimental value of the buoyancy force with the weight of the displaced water by calculating % Difference = ABS(Value 1 – Value 2)/AVERAGE(Value 1,Value 2).

$W_{in\ air}$ (N)	$W_{in\ water}$ (N)	F_B (N)	$m_{without\ water}$ (g)	$m_{with\ water}$ (g)	m_{water} (kg)	$F_{g,\text{water}}$ (N)	% Difference

Table 2: Replace this text with an appropriate caption.

Part 3. Density of a Liquid

The purpose of this experiment is to apply Archimedes' principle to experimentally determine the density of a liquid. For objects with constant cross-sectional areas (A) such as cylinders or cubes one can use the following equation:

$$F_B = \rho_{fluid} A d g$$

From this equation, you can see that the magnitude of the buoyant force is directly proportional to the portion of the object submerged in the liquid d. The graph F_B vs. d is a linear function with slope equal to $\rho_{\text{water}} A g$.

$$(\text{Slope}) = \rho_{fluid} A g$$

To find the fluid density experimentally, you will simulate dunking an object into a fluid, and graph the buoyant force vs, depth. The equation abouve can be re-arrange to express the density in terms of the slope, cross-sectional area, and g.

$$\rho_{fluid} = \frac{(Slope)}{(A \times g)}$$

1. Open a link to Physlet Physics, 3rd Edition, Exploration 14.2 Buoyant Force: https://www.compadre.org/Physlets/fluids/ex14_2.cfm
2. Set the mass to 0.375 kg and leave the density of fluid set to 700 kg/m^3.
3. Press the "initialize and play" button. Observe the motion of the block while keeping an eye on the value of the magnitude of the Tension in the string displayed at the top of the simulator. Notice that as the block is getting deeper in fluid, the force is decreasing indicating the decrease in weight of the block.
4. Press the "reset" button to get the block out of the liquid. At this setting, the block is at depth zero and the magnitude of tension in the string equals the magnitude of gravity.
5. In Table 3, record the value of force given by the simulator as both, the force of tension (F_T) and the force of gravity (F_g) for the depth zero. The magnitude of the gravitational force will remain the same for all depths, so just copy it down.
6. Click on the "step>>" button until the block is immersed 1.0 cm into the fluid. There are two ways to measure the depth:
   - the grid is 1 cm squares, and you can eyeball the depth or place a paper/plastic straight edge against your screen
   - If you click and hold the mouse on the screen (may not work with touch screens), a crosshair tool displays the x and y coordinates. Subtract the y coordinates of the top of the water and the bottom of the box.
7. Record the new depth and the magnitude of the tension force at this depth.

8. Repeat steps 6 and 7 for a total of at least 5 different non-zero depths.
9. For each depth, calculate the magnitude of the buoyant force as the difference between the magnitudes of gravitational and tension forces.
10. **Plot** the buoyant force F_B versus the submerged depth, d.
11. Determine the slope of the graph and record it in Table 4.
12. The base of the block in the simulation is a square of 5cm × 5cm. Calculate the cross-sectional area in m^2 (convert cm to m) and record it in Table 4.
13. Use the slope and the calculated area of the base (in m^2) to compute the experimental value of the density of the liquid.
14. Compare the experimental value of the density with the actual value displayed in the simulation by calculating the % Error.

Depth d (m)	**F_g (N)**	**$F_{T,\text{water}}$ (N)**	**F_B (N)**

Table 3: (Students should create an appropriate caption)

Slope of F_B vs. d (N/m)	
Area of base of block (m^2)	
Experimental Liquid Density (kg/m^3)	
Set Liquid Density (kg/m^3)	
% Error	

Table 4. (Students should create an appropriate caption)

Repeat the experiment with a different object and liquid:

- In the simulation, set the object mass to 0.25 kg and the liquid density to 1000 kg/m^3.
- Press “initialize and play” and continue the from step 3
- Label the new tables as 3a and 4a.

Requirements for the Lab 8 Formal Report (also consult the rubric):

- The Abstract should be written last BUT be at the front of the report
- The **Main Body** of the report must address the following
  - The **introductory** should have a background information about the nature of buoyant force and how it is reflected in Archimedes Principle.
  - The **methods** should explain:
    1. How the data was collected and calculated for Table 1 (what tool was used to measure the diameter of the ball and how this measurement was used for the calculation of the volume of the ball; how the displaced water was collected and the volume of the displaced water was calculated).
    2. How the data was collected and calculated for Table 2 (what tool was used to measure the weight of the ball in the air/water and how the buoyant force was computed from these measured values; explain what was measured to compute the weight of the displaced water and how these measurements were done)
    3. How the data was collected and calculated for Tables 3 and 3a including a comment on how the manipulation of the submerged depth was reflected in the measured values of the buoyant force.
    4. How the data from Tables 3 and 3a was analyzed including interpretation of the trendlines (how the slope of the graph was used to calculate the value of the density)

  - The **discussion** should quantitatively present how well the data from Table 1 and 2 supports Archimedes Principle (state what was expected and if it was achieved) and use tables 4 and 4a to discuss how Archimedes Principle was used in calculations of densities (quantitatively compare the experimental and accepted values of the densities).

- The **data section** must include:
  - 6 Tables (labeled and captioned)
  - 2 Graphs (titled, axis labels, units, labeled and captioned)